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# A Study on Realizing Relatively Prime Domination: Elucidation of Switching Tactic in Pentagon Snake Graphs

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Abstract

In This paper familiarizes the concept of a relatively prime dominating set in graphs, defined as a dominating set with at least two vertices where the degrees of any pair of vertices within the set are relatively prime. The minimum cardinality of such a set, denoted as $\gamma_{rpd}$(G), is investigated. Moreover, the paper deliberates vertex switching, a transformation that amends edges based on a designated vertex subset. The primary focus of this research is to determine the relatively prime domination number for various pentagon snake graph structures. It is shown that for standard pentagon snake graphs, $\gamma_{rpd}(P(S_n))$. is either 3 or 5. For alternate and alternate double pentagon snake graphs, $\gamma_{rpd}(A(P(S_n)))$ and $\gamma_{rpd}(A(D(P(S_n))))$ are both established to be 3.

**Keywords:** *Relatively Prime Dominating set, Switching Vertex, Pentagon Snake Graph, Alternate Pentagonal Snake Graph, Alternate Double Pentagonal Snake Graph.*



## 1. INTRODUCTION

Graph theory provides a powerful framework for demonstrating and analysing relationships between objects. Within this domain, the concept of domination has emerged as a fundamental tool for understanding connectivity and influence. However, recent research has sought to refine and extend the traditional notion of domination by introducing additional constraints and properties. One such refinement is the concept of "relatively prime domination," introduced by Jayasekaran and Jancy Vini [9]. This concept adds a number-theoretic dimension to domination, requiring that the greatest common divisors of the degrees of any two vertices within the dominating set must be relatively prime (i.e., their greatest common divisor is 1). This added constraint opens up new paths for exploring graph properties and their applications in areas where numerical relations between vertices are important. In this paper, we delve into the relatively prime domination of a specific class of graphs known as pentagon snake graphs. Furthermore, we integrate the technique of "vertex switching," a graph modification operation introduced by Lint and Seidel [2], which involves tactically altering the edges between vertex subsets and their complements. By applying vertex switching to pentagon snake graphs, we aim to investigate how these modifications affect the relatively prime domination number. This research seeks to provide a deeper understanding of the interplay between graph structure, number-theoretic properties, and vertex switching, eventually contributing to the broader field of domination theory. This research seeks to provide a deeper understanding of the interplay between graph structure, number-theoretic properties, and vertex switching, ultimately contributing to the broader field of domination theory.

## 2. DEFINITION

- **Definition 1.** The **Pentagon snake** $P(S_n)$ is obtained from the path $P_n$ by replacing each edge of the path by a pentagon $C_5$.

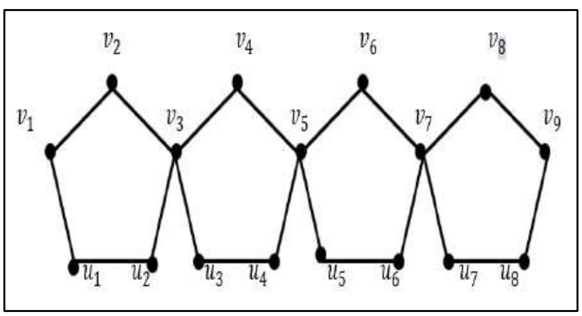


**Fig-1**-Pentagon Snake Graph

- **Definition 2.** An Alternate Pentagonal Snake $A(P(S_n))$ is obtained from a path $u_1, u_2, \dots u_n$, joining $u_i$ and $u_{i+1}$ to two new vertices $v_i, w_i$ and by joining $v_i$ and $w_i$ to a new vertices $x_i$ respectively.

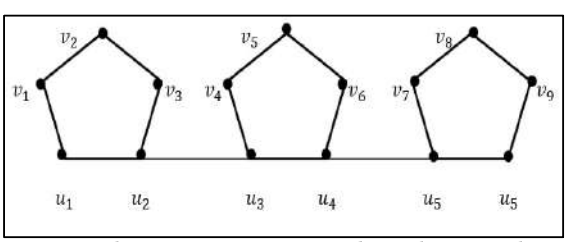


**Fig-2**-Alternate Pentagonal Snake Graph

- **Definition 3.** An Alternate Double Pentagonal **Snake** $A(D(P(S_n)))$ is obtained from two alternative pentagonal snakes that have a common path.

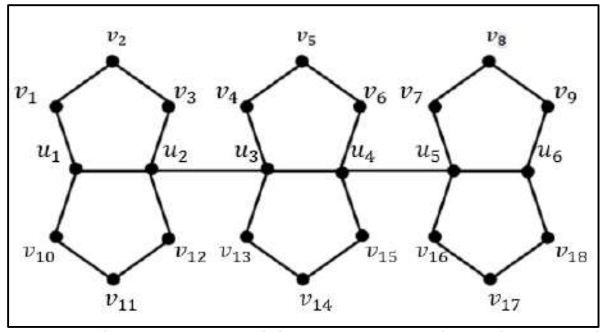


**Fig- 3**: Alternate Double Pentagonal Snake Graph

- **Definition 4.**

Let $G$ be a non-trivial graph. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices $u$ and $v$ in $S$ that $(\deg u, \deg v) = 1$. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$.

## 3. SWITCHING IN GRAPH

Switching in graphs was introduced by Lint and Seidel. For a finite undirected graph $G(V, E)$ and a subset $\sigma \subseteq V$, the switching of $G$ by $\sigma$ is defined as the graph $G^{\sigma}(V, E)$ which is obtained from $G$ by removing all edges between $\sigma$ and its complement $V - \sigma$ and adding as edges all non-edges between $\sigma$ and $V - \sigma$. For $\sigma = \{v\}$, we write $G^v$ instead of $G^{\{v\}}$ and the corresponding switching is called as vertex switching.

## 4. MAIN RESULTS

- **Theorem 1:** For any pentagonal snake graph $G$ with $p = 4n + 1$ vertices, where $n \geq 2$ there exists a relatively prime dominating set of size 3 or 5.

**Proof:**

Let $G$ be a pentagonal snake graph with $p$ vertices, where $p = 4n + 1, n \geq 2$. Let the vertices in the path be $v_1, v_2, \dots v_m$, where $v_1$ and $v_m$ denote the initial and end vertex respectively. The degree of each internal path vertex is 2 or 4; the degree of

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initial and end vertex is 2; the degree of vertices in the pentagon is 2. Let $v$ be any vertex in $G$. We have the following cases.

**Case (i):** $v$ is any vertex from $\{u_1, u_2, \dots u_{m-1}\}$.

Without loss of generality, let $v = u_i, i = 1,2, \dots m-1$. Then $d(v) = p - 3$. This vertex covers all the vertices of $G^v$ except the vertex which is adjacent to it. $i.e.$) $v_i$ and $u_{i+1}$. Then $d(v_i) = 1$ if $v_i$ is an initial vertex, otherwise 3 and $d(u_{i+1}) = 1$. Hence, the relatively prime dominating set is $\{v_i, u_i, u_{i+1}\}$ and $\gamma_{rpd}(G^v) = 3$. Suppose, if we take the vertex in between the pentagon and $d(v)$ is a multiple of 3. Then we cannot take the vertices adjacent to it together. So, we consider the vertices which are adjacent to $v_i$ and $u_{i+1}$. To cover the vertices $v_i$ and $v_{i+1}$ we must take the vertices $v_{i-1}$ and $u_{i+2}$. But, either of the one vertex has the degree multiple of 3. It is not possible to take these vertices. Thus, the relatively prime dominating set does not exist.

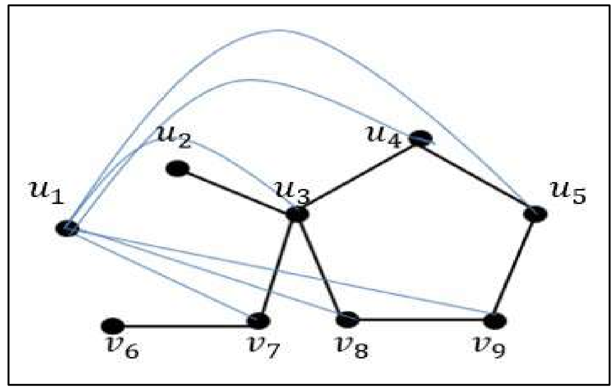
**Case (ii):** $v$ is an initial or end vertex.

Without loss of generality, let $v = v_i$, where $i = 1$ or $m$. Then $d(v_i) = p - 3$ in $G$. This vertex covers all the vertices of $G^v$ except two vertices namely $u_i$ and $v_{i+1}$. Here, $d(u_i) = 1$ and $d(v_{i+1}) = 1$. The set $\{u_i, v_i, v_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v) = 3$.

**Case (iii):** $v$ is any internal path vertex. Without loss of generality, let $v = v_i$, where $i = 2,3, \dots m-1$. Then $d(v_i) = p - 3$ in $G$. This vertex covers all the vertices of $G^v$ except two vertices namely $v_{i-1}$ and $v_{i+1}$. Here, $d(v_{i-1}) = 1$ and $d(v_{i+1}) = 3$. The set $\{v_{i-1}, v_i, v_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v) = 3$. Otherwise, the relatively prime dominating set is $0$ or $5$.

**Illustration**

For $n = 2$ and $p = 4n + 1 = 9$




Here, $n = 2$ and $p = 4n + 1 = 9$.

Let the switching vertex is $v_1$.

Thenclearly,

$d(u_1) = 6,\ d(u_2) = 1,\ d(u_3) = 4,\ d(u_4) = 3,$
$d(u_5) = 3,\ d(v_6) = 1, d(v_7) = \cdots d(v_9) = 3$

The relatively prime dominating set is $\{u_1, u_2, v_6\}$.

Hence, $\gamma_{rpd}(G^v) = 3$.

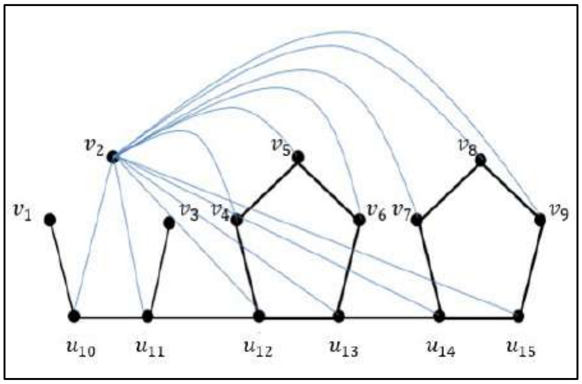
❖ **Theorem 2:** For any alternate pentagonal snake graph $G$ with $p = 5n$ vertices, where $n \geq 2$ there exists a relatively prime dominating set of size 3.

**Proof:**

Let $G$ be an alternate pentagonal snake graph with $p$ vertices, where $p = 5n, n \geq 2$. Let the vertices in the path be $u_1, u_2, \dots u_{m-2}$, where $u_1$ and $u_{m-2}$ denote the initial and end vertex respectively. Let $v_1, v_2, \dots v_m$ be the vertices in the pentagon. Clearly, the degree of initial and end vertex is 2; the degree of internal path vertex is 3. Also, $d(v_1) = d(v_2) = \cdots = d(v_m) = 2$. Suppose, if we take any vertex $v$, it covers all the vertices except the vertex adjacent to it.

Also, the adjacent vertex does not have the multiple of $d(v)$. Hence the relatively prime dominating set is $\{v_i, v_{i+1}, v_{i+2}\}$ and the relatively prime domination number is 3.

**Illustration**




Here, $n = 3$ and $p = 5n = 15$.

Let the switching vertex is $v_2$.
Thenclearly,
$d(v_1) = 1,\ d(v_2) = 12,\ d(v_3) = 1,\ d(v_4) = \cdots = d(v_9) = 3,\ d(u_{10}) = d(u_{15}) = 3,\ d(u_{11}) = \cdots d(u_{14}) = 4$

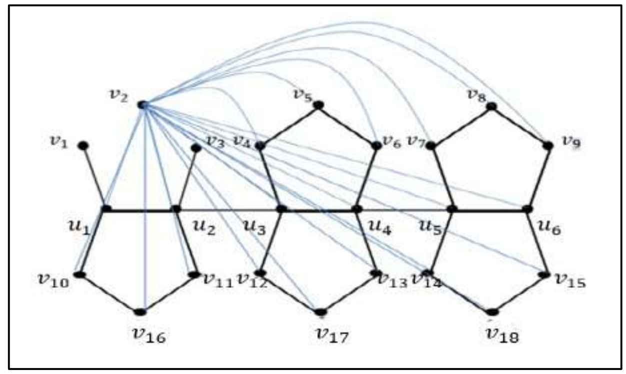
The relatively prime dominating set is $\{v_1, v_2, v_3\}$.
Hence, $\gamma_{rpd}(G^v) = 3$.

❖ **Theorem 3:** For any double alternate pentagonal snake graph $G$ with $p = 8n$ vertices, where $n > 2$ there exists a relatively prime dominating set of size 3.

**Proof:**

Let $G$ be a double alternate pentagonal snake graph with $p$ vertices, where $p = 8n, n \geq 2$. Let the vertices in the path be $u_1, u_2, \ldots u_k$, where $u_1$ and $u_k$ denote the initial and end vertex respectively. Let $v_1, v_2, \ldots v_m$ be the vertices in the upper pentagon and $v_{m+1}, v_{m+2}, \ldots v_s$ Clearly, the degree of initial and end vertex is 3; the degree of internal path vertex is 4. Also, $d(v_1) = d(v_2) = \cdots = d(v_s) = 2$. Suppose, if we take any vertex $v$, it covers all the vertices except the vertex adjacent to it. Also, the adjacent vertex does not have the multiple of $d(v)$. Hence the relatively prime dominating set is $\{u_{i+1}, v_i, v_{i+1}\}$ and the relatively prime domination number is 3.

**Illustration**




Here, $n = 3$ and $p = 8n = 24$.

Let the switching vertex is $v_2$.
Thenclearly,
$d(v_1) = 1,\ d(v_2) = 25,\ d(v_3) = 1,\ d(v_4) = \cdots = d(v_{15}) = 3,\ d(u_1) = d(u_6) = 4,\ d(u_2) = \cdots = d(u_5) = 5$

The relatively prime dominating set is $\{v_1, v_2, v_3\}$.
Hence, $\gamma_{rpd}(G^v) = 3$.

## 5. CONCLUSION

The verdicts of this research subsidize to a deeper understanding of these specific graph structures and offer a underpinning for further exploration of relatively prime domination in other graph families. The acumens gained may have insinuations for network design, where the control and interrelationships of nodes are crucial, or in other applications are relevant. Furthermore, the methodology employed in this study, which combines graph-theoretic analysis with the concept of relatively prime degrees, provides a valuable agenda for future research in related areas.

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