ISSN: 2583-7354

International Journal of Emerging Knowledge Studies

Publisher's Home Page: https://www.ijeks.com/

Fully Open Access

Research Paper

On NSBc Continuous Functions in Nano Topological Spaces

R.Raman[1], S.Pious Missier[2], E.Sucila[3]

[1]Research Scholar, (Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti, India.

[2]Head & Associate Professor(Rtd), Department of Mathematics, Don Bosco College of Arts and Science, Keela Eral, Thoothukudi, Tamilnadu, India.

[3]Associate Professor, Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti, India.

DOI: https://doi.org/10.70333/ijeks-03-12-016

*Corresponding Author: rramanram1988@gmail.com

Article Info:- Received : 26 November 2024 **Accepted** : 25 December 2024 **Published** : 30 December 2024

Abstract

This paper introduces a new class of continuous functions, termed nano Sb-continuous functions (Nº-cts), in nano topological spaces (NºT spaces). This study explores their fundamental properties, characterizations, and relationships with other generalized forms of continuity. We define and analyze the structural behavior of Nº-Sb continuous functions, extending the existing framework of nano topology. Several new propositions are established to demonstrate their role in preserving topological properties. Furthermore, the relationships between Nº-semi open sets, Nº-b-open sets, and Nº-Sb-continuous functions are discussed in detail. The results provide a foundation for further studies in generalized continuity within nano topological structures, offering new insights into the field of mathematical topology. The research contributes to advancing theoretical developments and potential applications in areas that utilize nano topology, such as fuzzy logic, rough set theory, and computational topology.

Keywords: *Nº-Semi Open-set, Nº-b-Open-set, Nº-Sb-Continuous Function.*

1. INTRODUCTION

The study of nano topology was started by M. Lellis Thivagar et al [8] with regard to a subset X of a universe that is described in terms of lower, upper and boundary approximations of X. He additionally described Nº-interior (briefly, Nº-int) and Nº-closure (briefly, Nº-cl) in NºT space. Andrijevic [1] presented and studied a category of generalized open sets in a topological space referred to as b-open sets. Further C. Indirani et al [4] created and studied Nº-b-open sets (Nº bo sets) in nano topological spaces (NºT Spaces). Bc open sets were first introduced in topological spaces by Hariwan Z. Ibrahim [6].

In this research paper, a new kind of Nº cts-functions referred to as Nº-Sbc cts-functions in NºT spaces is presented and its characteristics are examined.

2. PRELIMINARIES

Definition 2.1 [8] Let ŨN be a non-empty finite set of elements called the universal set and $\mathcal{R}$N represents an equivalence relation on ŨN referred as the indiscernibility relation. Elements in the same equivalence class are known as indiscernible with each another. The pair (ŨN, $\mathcal{R}$N) is known as the approximation space. Let X ⊆ ŨN.

(i) The lower approximation of X with respect to $\mathcal{R}_N$ is the collection of all elements which can be for certain classified as X with respect to $\mathcal{R}_N$ and it is represented by $L_{\mathcal{R}}$(X). That is, $L_{\mathcal{R}}(X) = \cup_{X \in \tilde{U}}\{ \mathcal{R}_x : \mathcal{R}_x \subseteq X \}$ where $\mathcal{R}_x$ is the equivalence class determined by X ∈ Ũ.

(ii) The upper approximation of X with respect to $\mathcal{R}_N$ is the collection of all elements, which can be possibly classified as X with respect to $\mathcal{R}_N$ and it is represented by $U_{\mathcal{R}}$(X).That is $U_{\mathcal{R}}(X) = \cup_{X \in U} \{\mathcal{R}_x : \mathcal{R}_x \cap X \neq \emptyset \}$.

(iii) The boundary region of X with respect to $\mathcal{R}_N$ is the set of all elements, that can be classified neither as X nor as not-X with respect to $\mathcal{R}_N$ and it is represented by $B_{\mathcal{R}}$(X).That is, $B_{\mathcal{R}}(X) = U_{\mathcal{R}}(X) - L_{\mathcal{R}}(X)$.

Definition 2.2 [8] **Let** $\tilde{U}_N$ be an universal set and $\mathcal{R}_N$ represent an equivalence relation on $\tilde{U}_N$. Then $№_{\mathcal{R}}(X) = №^T = \{\tilde{U}_N, \emptyset_N, L_{\mathcal{R}}(X), U_{\mathcal{R}}(X), B_{\mathcal{R}}(X)\}$ where $X \subseteq \tilde{U}_N$. Then, $№_{\mathcal{R}}(x)$ satisfies the axioms listed below.

(i) $\tilde{U}_N$ and $\emptyset_N \in №_{\mathcal{R}}(x)$.

(ii) The union of the elements of any sub collection of $№_{\mathcal{R}}(x)$ is in $№_{\mathcal{R}}(x)$.

(iii) The intersection of the elements of any finite sub- collection of $№_{\mathcal{R}}(x)$ is in $№_{\mathcal{R}}(X)$. Therefore, $№_{\mathcal{R}}(x)$ is a topology on $\tilde{U}_N$ named as №- topology ($№^T$) on $\tilde{U}_N$ with respect to X. We call ($\tilde{U}_N$, $№_{\mathcal{R}}(x)$) (or) ($\tilde{U}_N$, $№^T$) as the №TS. The elements of $№^T$ are called №-open sets (briefly, №-open sets). The complement of nano open sets are №- closed sets (briefly, №-closed sets).

Example 2.3 [8] Let $\tilde{U}_N = \{w_1, w_2, w_3, w_4\}$ with $\tilde{U}_N / \mathcal{R}_N = \{\{w_1\},\{w_2\},\{w_3, w_4\}\}$ and $X = \{w_1, w_3\} \subset \tilde{U}_N$. Then $N_{\mathcal{R}}(x) = №^T = \{\tilde{U}_N, \emptyset, \{w_1\}, \{w_3, w_4\}, \{w_1, w_3, w_4\}\}$.

Remark 2.4 [8] If $N_{\mathcal{R}}(x) = №^T$ is the nano topology on $\tilde{U}_N$ with respect to X and B_N is a nano subset of №T Space ($\tilde{U}_N$, $№^T$), then $B_N = \{\tilde{U}_N, L_{\mathcal{R}}(X), B_{\mathcal{R}}(X)\}$ is referred to as the basis for $N_{\mathcal{R}}$(X).

Definition 2.5 [8] If ($\tilde{U}_N$, $№^T$) is a NTS with respect to X where $X \subseteq \tilde{U}_N$ and if A_N is a nano subset in №TS and if $A_N \subseteq \tilde{U}_N$, then

(1) The№- interior of A_N is defined as the union of all №-O subsets of A_N and it is denoted by № int (A_N). That is, №- int (A_N) is the largest №-O subset of A_N.

(2) The №- closure of A_N is defined as the intersection of all №-C sets containing A_N and it is denoted by №- cl (A_N). That is, №- cl (A_N) is the smallest №-C set containing A_N.

Definition 2.6: Let ($\tilde{U}_N$, $№^T$) be a №TS and $A_N \subseteq \tilde{U}_N$. Then A_N is called

(1) №-semi open set (№-SO set) [8] if $A_N \subseteq$ № cl[№ int (A_N)] and №- semi- closed (№-SC set) [7] if №-int [№-cl (A_N)] $\subseteq A_N$.

(2) №-pre open set (№-PO set) [8] if $A_N \subseteq$ №-int [№- cl (A_N)] and №- pre-closed (№-PC set) [7] if №- cl [№- int (A_N)] $\subseteq A_N$.

(3) №- θopen set (№-θ-OS) [3] if for each $x \in A_N$, there exists a №- open set (№-OS) G_N such that $x \in G_N \subseteq$ №- cl (G_N) $\subseteq A_N$.

(4) №-$\boldsymbol{\theta}$-semiopen (№-$\boldsymbol{\theta}$-SO) [3] if for each $x \in A_N$, there exists a №-semi open set (№-SO set) G_N such that $x \in G_N \subseteq$ № -cl (G_N) $\subseteq A_N$.

(5) №-regular open set (№-RO set) [8] if A_N = №-int [№-cl (A_N)] and №- regular-closed (№-RC set) [7] if № -cl [№- int (A_N)] = A_N.

№-SO($\tilde{U}_N$, X), №-PO($\tilde{U}_N$, X), №-$\boldsymbol{\theta}$O($\tilde{U}_N$, X) , №-$\boldsymbol{\theta}$SO($\tilde{U}_N$, X)and №-RO($\tilde{U}_N$, X) respectively denotes the families of all nano semi-open(№-SO), nano pre-open(№-PO), nano $\boldsymbol{\theta}$-open(№-$\boldsymbol{\theta}$O) nano $\boldsymbol{\theta}$ semi-open(№-$\boldsymbol{\theta}$SO) and nano regular-open(№-RO) subsets of ($\tilde{U}_N$, X) .

Definition 2.7 Let ($\tilde{U}_N$, $N_{\mathcal{R}}$(X)) and ($\tilde{V}_N$, $N_{\mathcal{R}}$ (y)) be two №TS. A function η: ($\tilde{U}_N$, $N_{\mathcal{R}}$(X)) → ($\tilde{V}_N$, $N_{\mathcal{R}}$ (y)) is called

(1) №-cts [11] if η^{-1} (B_N) is № open set (№-OS) in $\tilde{U}_N$ for each № OS B_N in $\tilde{V}_N$.

(2) N№-semi-cts [10] if η^{-1} (B_N) is № semi OS (№-SO) in $\tilde{U}_N$ for each № OS B_N in $\tilde{V}_N$.
(3) №-pre-cts [10] if η^{-1} (B_N) is № pre OS (№-PO) in $\tilde{U}_N$ for every № OS BN in $\tilde{V}_N$..
(4) № b-cts [4] if η^{-1} (B_N) is № b-OS (№-b-OS) in $\tilde{U}_N$ for each № OS B_N in $\tilde{V}_N$.

Definition 2.8 [3] Let us consider №TS ($\tilde{U}_N$, №T) with the set $A_N \subseteq \tilde{U}_N$. Then the set A_N is known as Nano-b-open set (in brief, № b-O set) if $A_N \subseteq$№ cl (№ int (A_N))∪№ int (№ cl (A_N)). The complement of the № b-OS is known as nano- b-CS (briefly, № b-C set).

Example 2.9 [3] Let $\tilde{U}_N$ = {ɷ1, ɷ2, ɷ3, ɷ4} with $\tilde{U}_N$ / R ={ɷ1}, {ɷ3}, {ɷ2, ɷ4} and X = {ɷ1, ɷ2}. Then №T = { $\tilde{U}_N$, $\emptyset_N$, {ɷ1}, {ɷ1, ɷ2, ɷ4}, {ɷ2, ɷ4}} and № b-open sets are $\tilde{U}_N$, $\emptyset_N$, {ɷ1}, {ɷ2}, {ɷ4},{ɷ1, ɷ2},{ɷ1, ɷ3},{ɷ1, ɷ4},{ɷ2, ɷ4},{ɷ1, ɷ2, ɷ3},{ɷ1, ɷ2, ɷ4},{ɷ2, ɷ3, ɷ4}.

3. NANO SB^C-CONTINUOUS FUNCTIONS

Definition 3.1 Let AN be a subset of a №-TS (ŨN, №T). Then the set AN is called as № b^C-O set if for every $\varkappa \in$ AN $\in$ №-BO(ŨN), there exists a №-CS HN such that $\varkappa \in H$N $\subset$ AN.

The family of all nano b^C-open subsets (№ b^C-OS) of a №-T space (ŨN, №T) is denoted by
№ B^C-O (ŨN, №T) (briefly № B^C-O (ŨN).

Example 3.2 Let Ũ = {ɷ1, ɷ2, ɷ3, ɷ4} with Ũ/R ={{ɷ1},{ɷ3},{ɷ2, ɷ4}} and X ={ɷ1, ɷ2}. Then τN = {Ũ,$\emptyset$, {ɷ1},{ɷ1, ɷ2, ɷ4},{ɷ2, ɷ4}}. Then the №-closed sets are Ũ, φ, {ɷ2, ɷ3, ɷ4},{ɷ3} and {ɷ1, ɷ3}.Then the collection of all № b-open sets are №-bO(Ũ) = {Ũ,$\emptyset$, {ɷ1},{ɷ2},{ɷ4},{ɷ1, ɷ2},{ɷ1, ɷ3},{ɷ1, ɷ4},{ɷ2, ɷ4},{ɷ1, ɷ2, ɷ3},{ɷ1, ɷ2, ɷ4},{ɷ2, ɷ3, ɷ4}} and №-B^{C}O(Ũ) = {Ũ, $\emptyset$, {ɷ2, ɷ3, ɷ4 },{ɷ2, ɷ3}}.

Definition 3.3 A function η: (ŨN, N_R(X)) → (ṼN, N_R (y)) with respect to X and y respectively is called № Semi b^C-continuous if for each №-SO set BN of y containing η(x) , there exists a №-B^C- OS AN of X containing x such that η(AN) $\subset$ №-cl (BN). If η is №-SB^C-continuous at every point of ŨN , then it is called №-SB^C-cts .

Example 3.4 Let ŨN = {ɷ1, ɷ2, ɷ3, ɷ4} with ŨN /R ={{ɷ1,ɷ2},{ɷ3}, ɷ4}} and X ={ɷ1, ɷ3}. Then NT
N_R(X) = { ŨN ,$\emptyset$, {ɷ3},{ɷ1, ɷ2, ɷ3},{ɷ1, ɷ2}}. №-CS (X) = {$\emptyset$, ŨN ,{ɷ1, ɷ2, ɷ4}, {ɷ4}, {ɷ3, ɷ4}}.Then №-bO(X) = { ŨN ,$\emptyset$, {ɷ1},{ɷ2},{ɷ4},{ɷ1, ɷ2},{ɷ1, ɷ3},{ɷ1, ɷ4},{ɷ2, ɷ4},{ɷ1, ɷ2, ɷ3},{ɷ1, ɷ2, ɷ4},{ɷ2, ɷ3, ɷ4}}. №-b^{C}O(X) = { ŨN, $\emptyset$, {ɷ2, ɷ3, ɷ4 },{ɷ2, ɷ3}}. Let ṼN = { z1, z2, z3, z4} with ṼN /$\emptyset$ ={{z1},{z3},{z2, z4}} and y ={ z1, z2}. Then N_R(y) = { ṼN, $\emptyset$, {z1},{z1, z2, z4},{z2, z4}}. №-SO(y) ={ṼN ,$\emptyset$, {z1},{z1, z3},{z2, z4}, { z1, z2, z4}, { z2, z3, z4 }}. Define η: (ŨN, N_R(X)) → (ṼN, N_R' (y)) by η(ɷ1) = z2, η(ɷ2) = z4,
η(ɷ3) = z1, η(ɷ4) = z3. Then η is №-Sb^C-cts.

Proposition 3.4 A function η : (ŨN, N_R($\varkappa$)) → (Ṽ, N_R (y)) is №-semi-bc-cts if and only if for each $\varkappa$ in X and each №-regular CS F_N of y containing η($\varkappa$), there exists a №-bc-OS A_N of X containing $\varkappa$ such that η(A_N) ⊂ F_N .

Proof: Suppose that every №-regular CS F_N of y containing η($\varkappa$), there exists a №-bc-OS A_N of x containing $\varkappa$ such that η(A_N) ⊂ F_N . Let B_N be a №-SO set in y containing η($\varkappa$), so №-Cl (B_N) = F_N is №-RC, then there exists a №-bc-OS A_N of x containing $\varkappa$ such that η(A_N) ⊂ F_N = №-Cl (B_N). Hence η is №-semi-bc-cts.

Conversely, let $\varkappa \in$ x and let F_N be any №-RC set of y containing η($\varkappa$). Since η is №-Sbc-cts, then there exists a №-bc-OS A_N of x containing $\varkappa$ such that η(A_N) ⊂ №-Cl (F_N) = F_N .

Proposition 3.5 A function η : (Ũ, N_R(x)) → (Ṽ, N_R (y)) is №-Sbc-cts if and only if for every $\varkappa \in X$ and each №-θ-semi-OS H_N of y containing η($\varkappa$), there exists a №-bc-open set G_N of X containing x such that η(G_N) ⊂ H_N .

Proof : Let $\varkappa \in X$ and Ṽ be any №-SO set of y containing η($\varkappa$). So №-cl(H_N) is a №- θ semi-open set of y containing η($\varkappa$). Then there exists a №-bc-open set G_N of X containing $\varkappa$ such that η(G_N) ⊂ №-cl-(H_N). Hence η is №-Sbc-continuous.

Conversely, let H_N be any №-θ semi-open set of y containing η($\varkappa$), then there exists a №-semi-open set B_N of y such that η($\varkappa$) ∈ B_N ⊂ №-Cl(B_N) ⊂ H_N. Since η is №-Sbc-continuous, then there exists a №-bc-open set G_N of X containing $\varkappa$ such that η(G_N) ⊂ B_N ⊂ H_N.

Proposition 3.6 : For a function η : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)), the following statements are equivalent.

1. η is №-Sbc-continuous.
2. For each $\varkappa \in X$ and each №-semi-open set H_N of y containing η($\varkappa$), there exists a №-bc-open set G_N in X containing $\varkappa$ such that η(G_N) ⊂ №-PCl(H_N).
3. For each $\varkappa \in X$ and each №-regular closed set F_N of y containing η($\varkappa$), there exists a №-bc-open set G_N in X containing $\varkappa$ such that η(G_N) ⊂ F_N.
4. For each $\varkappa \in X$ and each №-θ semi-open set H_N of y containing η($\varkappa$), there exists a №-bc-open set G_N in X containing $\varkappa$ such that η(G_N) ⊂ H_N.

Proof :

(1) ⇒ (2). Since №-Cl(H_N) = №-pCl(H_N) for every H_N ∈ №-SO(Ṽ, η($\varkappa$)).

(2) ⇒ (3). Let $\varkappa \in X$ and let F_N be any №-regular closed set F_N of y containing η($\varkappa$).

Then for each η($\varkappa$) ∈ F_N there exists a №-semi-open set H_N containing η($\varkappa$) such that H_N ⊂ F_N .Then there exists a №-bc-open set G_N in X containing $\varkappa$ such that η(G_N) ⊂ H_N ⊂ F_N .

(3) ⇒ (4). Let $\varkappa \in X$ and let H_N be any №-θ semi-open set of y containing η($\varkappa$). Then for each η($\varkappa$)∈ H_N, there exists a № regular closed set F_N containing η($\varkappa$) such that F_N ⊂ H_N. By (3), there exists a № bc-open set G_N in X containing $\varkappa$ such that η(G_N) ⊂ F_N ⊂ H_N.

(4) ⇒ (1). It is already proved in Theorem 3.5.

Proposition 3.7: For a function η : (Ũ, $N_R(x)$) → (Ṽ, N_R(y)), the following statements are equivalent.

1. η is № semi-bc-cts.
2. η −1 (№ Cl(BN)) is № bc-OS in X, for each №-SO set BN in y.
3. η −1 (№ Int(F N)) is № bc-CS in X, for each №-SC set F N in y.
4. η −1 (BN) is № bc-CS in X, for each №-RO set BN of y.
5. η −1 (F N) is № bc-OS in X, for each №-RC set F N of y.
6. η −1 (BN) is № bc-OS in X, for each №-θ SO set BN of y.
7. η −1 (F N) is № bc-CS in X, for each №-θ SC set F N of y.

Proof:

(1) ⇒ (2). Let B_N be any № semi-OS in y. To prove that η^{-1} (№ Cl(B_N)) is № bc-OS in X. Let $\varkappa \in \eta^{-1}$ (№ Cl(B_N)). Then η($\varkappa$) ∈ № Cl(B_N) and № Cl(B_N) is a № regular CS in y. Since η is № semi-bc-cts. Then by Theorem 3.4, there exists a № bc OS A_N of X containing x such that η(A_N) ⊂ № Cl(B_N), Which implies that x ∈ A_N ⊂ η^{-1} (№ Cl(B_N)). Therefore, η^{-1} (№ Cl(B_N)) is № bc OS in X.

(2) ⇒ (3). Let F_N be any № semi-CS of y. Then y \\$F_N$ is a№ semi-OS of y. By (2), η^{-1} (№ Cl(y \\$F_N$)) is № bc-OS in X and η^{-1} (№ Cl(y \\$F_N$)) = η^{-1}(y \\№ int(F_N)) = X \ η^{-1} (№ int(F_N)) is № bc-OS in X and hence η^{-1} (№ int(F_N)) is № bc-CS in X.

(3) ⇒ (4). Let B_N be any № regular open in y. Then B_N is № semi-closed in y and № Int(B_N) = B_N . By (3), η^{-1} (№ Int(B_N)) = η^{-1} (B_N) is № bc closed set in X.

(4) ⇒ (5). Let F_N be any№ regular closed set of y. Then y \\$F_N$ is № regular open set of y. By (4), η^{-1} (y \\$F_N$) is № bc-closed set in X and η^{-1} (y \\$F_N$ = X \ f−1(F_N). Therefore η^{-1} (F_N) is № bc-open set in X.

(5) ⇒ (6). It follows from the fact that any № θ semi-open set is a union of № regular closed sets.

(6) ⇒ (7). . It is entirely analogous to part (4) ⇒ (5) and the proof is obvious.

ISSN: 2583-7354

Proposition 3.8: For a function η : (Ũ, $N_R(x)$) → (Ṽ, $N_R(y)$), the following statements are equivalent.

1. η is № semi-bc-continuous.
2. № bc-Cl ($η^{-1}(B_N)$) ⊂ $η^{-1}$(№ Int(Cl(B_N)), for each № PO set B_N of y.
3. η −1(№ Cl(№ Int(FN))) ⊂ № bc Int(η −1(FN)), for each № PC set FN of y.

Proof :
(1) ⇒ (2). Let B_N be any № pre-open set of y. Then B_N ⊂ № Int(№ Cl(B_N)) and№Int(№Cl(B_N)) is № regular open set in y. Since η is № semi-bc continuous, by Theorem 3.7, (4), $η^{-1}$ № Int(№ Cl(B_N))) is № bc-closed set in X and hence we obtain that № bc Cl($η^{-1}$ (B_N)) ⊂ $η^{-1}$ (№ Int(№ Cl(B_N)).

(2) ⇒ (3). Let F_N be any № pre-closed set of y. Then Y\$F_N$ is № pre-open set of y and by (2), we have №-bcCl($η^{-1}$ (y \$F_N$)) ⊂ $η^{-1}$ (№ Int(Cl(y \$F_N$)) ⟺ X \ №-bc Int($η^{-1}$ (F_N) ⊂ $η^{-1}$ (y \№ Cl(№ Int(F_N))) ⟺ X \ №-bc Int($η^{-1}$ (F_N)) ⊂ X \ $η^{-1}$ (№ Cl(№ Int(F_N))). Therefore, $η^{-1}$ (№ Cl(№ Int(F_N))) ⊂ №-bc Int($η^{-1}$ (F_N)).

(3) ⇒ (1). Let B_N be any № RO set of y. Then Y\B_N is № pre-open set of y .Then we have № bc Cl($η^{-1}$ (B_N)) ⊂ $η^{-1}$ (№ Int(№ Cl(B_N)) = $η^{-1}$(B_N). Hence $η^{-1}$(B_N) is №-bc closed set in X and hence by theorem 3.7,(4), η is №-Sbc continuous.

Proposition 3.9: For a function η : (Ũ, $N_R(x)$) → (Ṽ, $N_R(y)$), the following statements are equivalent.

1. η is № semi-bc-continuous.
2. № bc-Cl ($η^{-1}(B_N)$) ⊂ $η^{-1}$(№ SCl(B_N)), for each № PO set B_N of y.
3. $η^{-1}$(№ SInt(F_N))) ⊂ № bc Int($η^{-1}$(F_N)), for each № PC set F_N of y.

Proof:
(1) ⇒ (2). Let B_N be any № pre-open set of y. Since №-SCl(B_N) = № Int(№ Cl(B_N) for each B_N of №PO(y), it follows that B_N ⊂ № Int(№ Cl(B_N)) and№Int(№Cl(B_N)) is № regular open set in y. Since η is № semi-bc continuous, by Theorem 3.7, (4), $η^{-1}$ № Int(№ Cl(B_N))) is № bc-closed set in X and hence we obtain that № bc Cl($η^{-1}$ (B_N)) ⊂ $η^{-1}$ (№ Int(№ Cl(B_N)).

(2) ⇒ (3). Let F_N be any № pre-closed set of y. Then Y\$F_N$ is № pre-open set of y and by (2), we have №-bcCl($η^{-1}$ (y \$F_N$)) ⊂ $η^{-1}$ (№ Int(Cl(y \$F_N$)) ⟺ X \ №-bc Int($η^{-1}$ (F_N) ⊂ $η^{-1}$ (y \№ Cl(№ Int(F_N))) Since №-SInt(F_N) = № Cl(№ Int(F_N) ⟺ X \ №-bc Int($η^{-1}$ (F_N)) ⊂ X \ $η^{-1}$ (№ Cl(№ Int(F_N))). Therefore, $η^{-1}$ (№ Cl(№ Int(F_N))) ⊂ №-bc Int($η^{-1}$ (F_N)).

(3) ⇒ (1). Let B_N be any № RO set of y. Then Y\B_N is № pre-open set of y .Then we have № bc Int($η^{-1}$ (B_N)) ⊂ $η^{-1}$ (№ cl(№ Int(B_N)) = $η^{-1}$(B_N). Hence $η^{-1}$(B_N) is №-bc closed set in X and hence by theorem 3.7,(4), η is №-Sbc continuous.

Proposition 3.10 : A function η : (Ũ, $N_R(x)$) → (Ṽ, $N_R(y)$) is № Sbc-continuous if and only if $η^{-1}$(B_N) ⊂ №-bc Int ($η^{-1}$(№ cl(B_N)) for each№ SO set B_N of y.

Proof : Let B_N be any № SO set of y. Then B_N ⊂ №-cl (B_N) and №-cl (B_N) is № RC set in y. Since η is №-Sbc continuous , by Theorem 3.7,(5), $η^{-1}$(№ cl(B_N)) is №-bc open set in X and hence $η^{-1}$(B_N) ⊂ $η^{-1}$(№ cl(B_N)) = №-bc Int ($η^{-1}$(№ cl(B_N)).
Conversely, let B_N be any № RC set of y. Then B_N is № SO set of y. Then $η^{-1}$(B_N) ⊂ №-bc Int ($η^{-1}$(№ cl(B_N)) = №-bc Int ($η^{-1}$(№ (B_N)). Then $η^{-1}$(B_N) is №-bc OS in X.Hence by Theorem 3.7, η is № semi-bc-continuous.

Proposition 3.11 : A function η : (Ũ, $N_R(x)$) → (Ṽ, $N_R(y)$) is № Sbc-continuous if and only if $η^{-1}$(B_N) ⊂ №-bc Int ($η^{-1}$(№ Pcl(B_N)) for each№ SO set B_N of y.

Proof : Let B_N be any № SO set of y. Then B_N ⊂ №-cl (B_N) and №-cl (B_N) is № RC set in y. Since η is №-Sbc continuous , by Theorem 3.7,(5) and Since №-Cl(B_N) = №-pCl(B_N), we have $η^{-1}$(№ Pcl(B_N)) is №-bc open set in X and hence $η^{-1}$(B_N) ⊂ $η^{-1}$(№ Pcl(B_N)) = №-bc Int ($η^{-1}$(№ Pcl(B_N)).

ISSN: 2583-7354

Conversely, let B_N be any № RC set of y. Then B_N is № SO set of y. Then η $^{-1}(B_N)$ ⊂ №-bc Int (η $^{-1}$(№ Pcl(B_N)) = №-bc Int (η $^{-1}$(№ (B_N)). Then η $^{-1}(B_N)$ is №-bc OS in X.Hence by Theorem 3.7, η is № semi-bc-continuous.

Proposition 3.12 : A function η : (Ũ, $N_R(x)$) → (Ṽ, N_R(y)) is № Sbc-continuous if and only if №-bc Cl (η $^{-1}$(№ Int(F_N)) ⊂ η $^{-1}(F_N)$ for each № SC set F_N of y.

Proof : Let F_N be any № SC set of y. Then № Int(№ cl (F_N)) ⊂ F_N . and №-cl (F_N) is № RC set in y. Since η is №-Sbc continuous , by Theorem 3.7, we have η $^{-1}$(№ cl(F_N)) is №-bc open set in X and hence №-bc Cl (η $^{-1}$(№ Int(F_N)) ⊂ η $^{-1}(F_N)$.
Conversely, let F_N be any № RC set of y. Then F_N is № SC set of y. Then №-bc Cl (η $^{-1}$(№ Int(F_N)) ⊂ η $^{-1}(F_N)$. Then η $^{-1}(F_N)$ is №-bc OS in X.Hence by Theorem 3.7, η is № semi-bc-continuous.

Proposition 3.13 : A function η : (Ũ, $N_R(x)$) → (Ṽ, N_R(y)) is № Sbc-continuous if and only if η $^{-1}(F_N)$ ⊂ №-bc Cl (η $^{-1}$(№ Int((№ Cl (F_N)) for each № PO set F_N of y.

Proof : Let F_N be any № PO set of y. Then F_N ⊂ № Int(№ cl (F_N)) and № Int(№ cl (F_N)) is № RO set in y. Since η is №-Sbc continuous , by Theorem 3.7, we have η $^{-1}$(№ Int((№ Cl (F_N)) is №-bc closed set in X and hence we have η $^{-1}(F_N)$ ⊂ η $^{-1}$(№ Int((№ Cl (F_N)) = №-bc Cl (η $^{-1}$(№ Int((№ Cl (F_N)) .

Proposition 3.14 : A function η : (Ũ, $N_R(x)$) → (Ṽ, N_R(y)) is № Sbc-continuous if η $^{-1}(B_N)$ ⊂ №-bc Cl (η $^{-1}$(№ Int(SCl (B_N)) for each № PO set B_N of y.

Proof : Let B_N be any № PO set of y. Then B_N ⊂ № Int(№ cl (B_N)) and № Int(№ cl (B_N)) is № RO set in y. Since η is №-Sbc continuous , by Theorem 3.7, we have η $^{-1}$(№ Int((№ Cl (B_N)) is №-bc closed set in X and hence we have η $^{-1}(B_N)$ ⊂ η $^{-1}$(№ Int((№ Cl (B_N)) = №-bc Cl (η $^{-1}$(№ Int((№ Cl (B_N)) .

References

Andrijevic, "On b-open sets in topological spaces", Math. Vesnik , 48(1), (1996), 59-64.

A.S.Mashhour, ME.Abd El Mousef and S.N.El peeb, "On Pre continuous and weak pre continuous mappings", proc.Math & Phys.Soc., Egypt. 53 (1982), 47-53.

A.Vadivel, J.Sathyaraj, M.Sujatha & M.Angayarkarasi, "Generalizations of nano θ closed sets in nano topological spaces", Journal of information and computational science, 9, (11), -2019.

C.Indirani, M.Parimala &S,Jafari, " On nano b -open sets in nano topological spaces ", Jourdan journal of mathematics and statistics(IJMS) 9(3),2016,173-184.

D.A.Mary and Arockiarani .A, " On Characterizations of nano rgb-closed sets in nano topological spaces ". Int.J.Mod.Eng.Res. 5(1) (2015) 68-76.

D.A.Mary and Arockiarani .A, " On b-open sets and b-continuous functions in nano topological spaces " , Int.J.Innov.Res Stud.,3(11) (2014) 98-116.

H.Z.Ibhrahim, "Bc -open sets in topological spaces", Advances in pure mathematics, 2013, 3,34-40.

K.F.Porter, "Regular open sets in topology", Int.Jour.ofMath.and Math. Sci.,2(1996) ,299-302.

M.Lellis Thivagar, C.Richard , "On nano Continuity," Math.Theory Model. 7(2013) 32-37.

Mohammed M.Khalaf and Kamal N.Nimer, " Nano Ps open sets and Ps continuity, " Int.J.of. contemp.Math.Sciences, Vol.10, pp.1-11, 2015.

Levine, "Semi open sets and semi continuity in Topological Spaces", Amer.Math.Monthly, 70(1963), 36-41.

P.Sathishmohan , V,Rajendran, P.K.Dhanasekaran and Brindha.S., "Further properties of nano pre T_0 space, nano pre T_1 space and nano pre T_2 spaces" , Malaya Journal of Mathematik, 7,(1),2019, 34-38.

Raad Aziz Al-Abdulla, Ruaa Muslim Abed, "On Bc open sets in Topological Spaces", International Journal of Science and Research, 3,(10), (2014).

Cite this article as: R.Raman, S.Pious Missier and E.Sucila, (2024). On NSBc Continuous Functions in Nano Topological Spaces, International Journal of Emerging Knowledge Studies. 3(12), pp.1035-1040.
https://doi.org/10.70333/ijeks-03-12-016